

# Inference Control in Statistical Databases

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- 1. Confidential Data: Legal Requirements and Societal Expectations (20 min)**
- 2. Data Aggregation (20 min)**
- 3. Statistical Databases: Inference Control (70 min)**
- 4. Conclusion (10 min)**

# 1. Confidential Data: Legal Requirements and Societal Expectations

**With larger databases and greater data collection capabilities, more and more confidential data are aggregated in data repositories**

**Technology: Unexpected and unknown capabilities**

**May clash with a society's expectations and ethics**

**The expectation of privacy: may be contradicted by new technological capabilities**

**Legal landscape: US vs. EU**

**The European Union has far more stringent privacy and confidentiality requirements than the US. EU requirements are very general (referring to all data that can be used to identify an individual) while US privacy provisions are sectoral (typically financial, medical, student data).**

**How can the confidentiality of personal data (an important aspect of privacy) be safeguarded? It may be legally required, or it may be good customer relations to do so.**

## 2. Data Aggregation

**Two entirely innocuous databases may be combined to identify uniquely an individual**

**Example:**

**Consider Date of Birth (DoB) and consider the US 5-digit zip code**

**The US has approximately 320M inhabitants (2013).**

**There are no more than 100 000 (5-digit) zip codes: on average, there are at least 3 200 inhabitants with the same zip**

**Assuming a cut-off at 80, there are about  $365 \times 80$  birth dates: on average, there are about 12 000 people with the same DoB**

**Therefore, each item in each of the two databases is entirely safe since there are on average thousands of individuals that correspond to any such item (DoB or zip)**

**However, combining DoB and zip identifies uniquely many, if not most individuals, since**

$$100\ 000 \times 365 \times 80 = 2\ 920\ 000\ 000 \gg 320M$$

**Note: This is a statistical argument; there is no guarantee that a specific combination will not be satisfied by more than one individual (e.g., twins living in the same household).**

## 3. Statistical Databases: Inference Control

### The notion of a statistical database

**Two different access mechanisms:**

**Ordinary database access**

**Access to statistics**

### Ordinary access

**Of primary concern are confidential data**

**Requires proper authorization, typically need to know**

**Involves access to individual entries**

**Example: A hospital's medical database**

**Treating physicians and nurses must have access to patient data,  
but only their patients', not of those of other healthcare providers**

## Access to statistics

**No access to individual entries, but rather to statistics over sets of individuals**

**E. g., averages, medians**

## Example continued:

**Medical database containing diagnoses and related information**

**A public health professional wants to design a campaign to educate people about HIV infections. It would be useful to be able to target the most susceptible population group within the area served by the hospital. Thus, it would be useful to know more about the “average person infected with HIV” so the campaign may be more specifically tailored to that population. This necessitates statistical queries.**

**Clearly, this public health official must not have access to any information about individuals in the database (e. g., is John Doe HIV-positive?).**

## **Confidential data, legal requirements to maintain confidentiality:**

**The use of statistical queries must guarantee confidentiality**

## **Inference control:**

**It must be impossible to use legitimate statistical queries in order to obtain access to individual confidential data**

**Of particular concern is combining various statistical queries so that access to individual entries is achieved**

**In practice, inference control is extremely difficult to attain**

**Extensive literature, for many approaches, restrictions, and models of statistical databases**



## Example

**Assume ordinary SQL queries**

**Each query involves a set of elements**

**Typical: Median, average, sum**

**Confidential information in statistical queries is numerical**

**Assume query type SUM:**

**A query defines a set of elements and returns the sum of the confidential information associated with all the elements in the set**

**In SQL queries, we can do or (forming the union of the underlying sets) and not (forming the complement of the underlying set)**

**Note that at the technical/implementation level, the queries will always be based on (sets of) individual entries; the difference between ordinary queries (on a need-to-know basis) and statistical queries is what is being divulged to the user who poses the query**

**In order to obtain a secure statistical database, we impose the following restriction:**

**A statistical query  $q$  is legal if and only if**

$$h \leq \text{NU}(q) \leq N-h$$

**$N$  is the total number of elements in the database,  $h$  is an arbitrary value, and  $\text{NU}(q)$  is the number of elements in the set underlying the query  $q$**

**In other words, a legal statistical query must match more than only a few entries (at least  $h$  entries) and may not match too many entries (at least  $h$  entries must not be involved in the query). The value  $h$  is a parameter that can be chosen depending on the preference of the database administrator.**

**Larger  $h$  == more paranoia, smaller  $h$  == less paranoia.**

**Thus there are two bad cases, when  $q$  is too small (matches not enough entries) and when  $q$  is too large (matches too many entries)**

**This requirement appears to be reasonable and eminently intuitive.**

**Unfortunately, it fails spectacularly to prevent inference control.**

**First, define a general tracker GT:**

**This is any query GT such that**

$$2h \leq \text{NU}(\text{GT}) \leq N-2h$$

**In other words, it must be a bit larger ( $2h$  instead of  $h$ ) than an ordinary legal statistical query and it must be a bit bigger ( $N-2h$  instead of  $N-h$ ) than such a query**

**Consider now an illegal query  $q_{\text{bad}}$ :**

**$q_{\text{bad}}$  may be either too small ( $\text{NU}(q_{\text{bad}}) < h$ ) or too large ( $\text{NU}(q_{\text{bad}}) > N-h$ ).**

**The expectation is that the value of  $q_{\text{bad}}$  cannot be determined.**

**This is wrong, as its value is obtained as follows:**

**First compute  $x := \text{SUM}(\text{GT}) + \text{SUM}(\underline{\text{not}}\text{GT})$**

$$\text{SUM}(q_{\text{bad}}) = \text{SUM}(q_{\text{bad}} \underline{\text{or}} \text{GT}) + \text{SUM}(q_{\text{bad}} \underline{\text{or}} \underline{\text{not}} \text{GT}) - x \quad q_{\text{bad}} \underline{\text{too small}}$$

$$\text{SUM}(q_{\text{bad}}) = 2x - [\text{SUM}(\underline{\text{not}} q_{\text{bad}} \underline{\text{or}} \text{GT}) + \text{SUM}(\underline{\text{not}} q_{\text{bad}} \underline{\text{or}} \underline{\text{not}} \text{GT})] \quad q_{\text{bad}} \underline{\text{too large}}$$

**In either case, one can verify that the necessary queries are legal statistical queries and that their combination yields the supposedly confidential information, the value of the illegal query  $q_{\text{bad}}$ . While one does not know beforehand why the query is illegal (too small or too large), only one of the two cases applies and only that case will yield legal queries.**

## **Upshot**

**Impossible to maintain the confidentiality of the information of individual entries**

**Similar results hold for virtually all type of restrictions that are imposed on the queries**

**There is a large body of literature to that effect.**

**Randomizing is a possible solution, except the responses are falsified (slightly)**

**In other words, instead of providing the true response, the actual response is slightly modified, either by adding a randomly selected element from the database or deleting one of the elements involved in the query, and then returning the statistic based on the modified set rather than the original set of entries. This change in the value of the response can be exploited to prevent reasonably well the disclosure of confidential information thereby providing inference control. However, some users find the randomization unacceptable.**

## 4. Conclusion

**With the advent of larger and more centralized databases, statistical information based on these databases will become more important.**

**Inference control is paramount when the information collected in these databases is considered confidential.**

**Confidentiality may be legally required (e. .g, medical, financial, student data) or the consumer may insist on it.**

**Most mechanisms designed to achieve inference control do not work.**